*Roll No. .....*

# 2002

## B. E. 3rd Semester (ECE)

## Examination – December, 2013

### MATHEMATICS-III

**'E' Scheme**

**Paper : Math-201(E)**

*Time : Three hours ]* *[ Maximum Marks : 100*

*Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.*

***Note :*** Attempt any ***five*** questions taking at least ***one*** from each Section. All questions carry equal marks.

### SECTION – A

**1.** (a) Find the Fourier expansion of the function $f(x) = x^2, -2 < x < 2$, and hence prove that :

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}$$

(b) If $f(x) = \begin{cases} 0, & -\pi < x > 0 \\ \sin x, & 0 < x < \pi \end{cases}$, prove that :

$$f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{\cos 2mx}{4m^2 - 1}.$$

Hence show that :

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots\ldots\infty = \frac{(\pi - 2)}{4}$$

**2.** (a) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as a Fourier integral. Hence evaluate :

$$\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$$

(b) Solve the integral equation :

$$\int_0^{\infty} f(x) \sin px \, dx = \begin{cases} 1, & 0 < p < 1 \\ 2, & 1 < p < 2 \\ 0, & p > 2 \end{cases}$$

## SECTION – B

**3.** (a) If $\tan(\theta + i\varphi) = \tan\alpha + i \sec \alpha$, show that :

(i) $e^{2\varphi} = \pm \cot \alpha/2$

(ii) $2\theta = (n + 1/2)\pi + \alpha$

(b) Determine the analytic function whose imaginary part is $\log(x^2 + y^2) + x - 2y$.

4. (a) Define an analytic function. State and prove the necessary and sufficient conditions for a function to be analytic.

(b) Evaluate :

$$\oint \frac{\sin z^2 + \cos z^2}{(z-1)^2(z-2)}\, dz$$

where C is the circle $|z| = 3$.

5. (a) Using Cauchy's integral formula, evaluate :

$$\oint \frac{\sin^2 z}{(z-\pi/3)^3}\, dz$$

around the circle $|z| = 1$.

(b) Evaluate $\int_0^{\pi} \frac{1}{a + b\cos\theta}\, d\theta$, where $a > b$.

## SECTION – C

6. (a) In a bolt factory, there are 4 machines A, B, C and D manufacturing 20%, 15%, 25% and 40% of the total output respectively. Of their outputs 5%, 4%, 3% and 2% in the same order, are defective bolts. A bolt is chosen at random from the factory's production and is found defective. What is the probability that the bolt was manufactured by machine A or machine D ?

(b) Fit a binomial distribution to the following frequency distribution :

| $x$ : | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f$ : | 13 | 25 | 52 | 58 | 32 | 16 | 4 |

**7.** A survey of 320 families with 5 children each revelled the following distribution :

| No. of Boys : | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|
| No. of Girls : | 0 | 1 | 2 | 3 | 4 | 5 |
| No. of Families : | 14 | 56 | 110 | 88 | 40 | 12 |

Is this result consistent with the hypothesis that male and female births are equally probable ?

**8.** Using Simplex Method to solve the following L P P. :

Maximum : $z = 3x_1 + 5x_2 + 4x_3$

Subject to :

$$2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0$$

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