Roll No. ....

24022

B. Tech. 3rd Semester (Electrical Engg.) Branch - 1 Examination – December, 2011

MATHEMATICS-III

Paper : Math-201-F

Time : Three hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in total, selecting *one* question from each Unit Q. No. 1 is ***compulsory***. All questions carry equal marks.

1. (a) Find the Fourier series of the function defined by :

$$f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ \pi, & 0 \le x, \pi \end{cases}$$

(b) Solve the integral equation :

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}$$

(c) Find regular function whose imaginary part is $\dfrac{x-y}{x^2+y^2}$.

(d) Evaluate $\int_c (z - z^3)\, dz$, where c is upper half of the circle.

(e) Evaluate $\int_c \frac{z^2 - z + 1)}{z - 1}\, dz$, where c is the circle $|z| = \frac{1}{2}$.

(f) If a random variable has a Poisson distribution such that $P(1) = P(2)$. Find mean of the distribution.

(g) The average marks in English of a sample of 100 is S1 with a SD of 6 marks. Could this have a random sample from a population with average marks 50 ?

(h) Intelligent tests given of two groups of boys and girls :

	Mean	S.D	size
Girls	75	8	60
Boys	73	10	100

Examine if the difference between mean stores is significant.

UNIT – A

2. (a) Show that for $-\pi \leq x \leq \pi$

$$\cos cx = \frac{\sin c\pi}{\pi}\left[\frac{1}{c} - \frac{2c\cos x}{c^2 - 1^2} + \frac{2c\cos 2x}{c^2 - 2^2} + \ldots\ldots\ldots\right.$$

where c is non-integral, hence deduce a that

$$\pi \operatorname{cosec}(c\pi) = \sum_{n=0}^{\infty} (-1)^n \left[\frac{1}{n+c} + \frac{1}{n+1-c}\right]$$

(b) Find the Fourier series to represent $f(x) = x^2 - 2$ when $-2 \leq x \leq 2$.

3. (a) Find the Fourier sine and cosine transform of the function x^{m-1}.

(b) State and prove convolution theorem for Fourier transforms.

UNIT – B

4. (a) If $C\tan(x+iy)=A+CB$ prove that

$$\tan 2x=\frac{2CA}{C^2-A^2-B^2}$$

(b) Determine the analytic function whose real part is $e^{2x}(x\cos 2y-y\sin 2y)$.

5. (a) If f(z) in a regular function of z, prove that $\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}\right)|f(z)|^2=4|f'(z)|^2$.

(b) If $f(\xi)=\int_c \frac{3z^2+7z+1}{z-\xi}$, where c is the circle $x^2+y^2=4$. Find the value of $f''(1-i)$.

UNIT – C

6. (a) Show that when $|z+1|<1$,

$$z^{-2}=1+\sum_{n=1}^{\infty}(n+1)(z+1)^n$$

(b) Evaluate $\int_0^{2\pi}\frac{d\theta}{1-2a\sin\theta+a^2}, 0<a<1$.

7. (a) The probability that a man aged 50 years will die within a year is 0.01125. What is the probability that out of 12 such men, at least 11 will reach their fifty first birthday ?

(b) Prove that mean deviation from the mean of a normal distribution is $\frac{4}{5}$ of its standard deviation.

UNIT – D

8. (a) Using simplex method

Maximize $z = x_1 + 2x_2$,

Subject to $2x_1 + x_2 \leq 8$,

$2x_1 + 3x_2 \leq 12$

$x_1, x_2 \geq 0$

(b) Obtain the dual of :

Maximize $z = 5x_1 + 3x_2$,

Subject to $x_1 + x_2 \leq 2$,

$5x_1 + 2x_2 \leq 10$,

$3x_1 + 8x_2 \leq 12$,

$x_1, x_2 \geq 0$

9. (a) The 9 items of a sample have the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these values differ significantly from the assumed mean 47.5.

(b) A certain stimulus administered to each of 12 patients resulted in the following increases of blood pressures : 5, 2, 8, –1, 3, 0, –2, 1, 5, 0, 4, 6. Can it be concluded that he stimulus will in general be accompanied by an increase of blood pressure ?