Roll No. ....

24022

B. Tech. 3rd Semester (Electrical Engg.)

Examination – December, 2012

MATHEMATICS - III

Paper : Math-201-F

Time : Three hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Questions No. 1 is ***compulsory***. Attempt total *five* questions with selecting ***one*** from each **Section**. All questions carry equal marks.

1. (a) Evaluate $\oint_c \frac{e^{-z}}{z+1} dz$, where c is the circle $|Z| = \frac{1}{2}$.

(b) State Residue theorem.

(c) If A and are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{2}$. Show that A and B are independent events.

(d) Define Poisson Distribution.

(e) If $f(x) = \left(\frac{\pi - x}{2}\right)^2$, $0 < x < 2\pi$, Find a_n

(f) Find the Fourier cosine transform of $f(x) = e^{-ax}$

(g) State Convolution Theorem for Fourier Transform.

(h) Prove that $\tan\left(i \log \frac{a-ib}{a+ib}\right) = \frac{2ab}{a^2 - b^2}$

SECTION – A

2. (a) Find the Fourier series of the Function.

$$f(x) = x \sin x, -\pi \le x \le \pi$$

Also deduce that : $\frac{1}{1.3} - \frac{1}{3.5} - \frac{1}{5.7} + \ldots\ldots = \frac{\pi - 2}{4}$

(b) If $f(x) = x, 0 < x < \frac{\pi}{2}$

$$= \pi - x, \frac{\pi}{2} < x < \pi$$

Show that :

(i) $f(x) = \frac{4}{\pi}\left\{\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} - \frac{\sin 5x}{5^2} - \ldots\ldots\ldots\right\}$

(ii) $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left\{\frac{\cos 2x}{1^2} - \frac{\cos 6x}{3^2} - \frac{\cos 10x}{5^2} - \ldots\ldots\right\}$

3. (a) Solve the integral equation

$$\int_0^\infty f(x) \sin px \, dx = \begin{cases} 1, & 0 < p < 1 \\ 2, & 1 < p < 2 \\ 0, & p > 2 \end{cases}$$

(b) Express the function $f(x) = \begin{cases} 1 \text{ For } |x| \le 1 \\ 0 \text{ For } |x| > 1 \end{cases}$ as a Fourier integral.

Hence Evaluate $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$

SECTION – B

4. (a) If $u = \log \tan(\pi/4 + \theta/2)$, prove that.

   (i) $\tanh u/2 = \tan \theta/2$

   (ii) $\cosh u = \sec \theta$

   (b) Derive Cauchy-Riemann equations in Polar form. Hence deduce that

$$\left(\partial^2 u / \partial r^2\right) + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\left(\partial^2 u / \partial \theta^2\right) = 0$$

5. (a) Define an analytic function. State and prove the necessary and sufficient conditions for a function to be analytic.

   (b) Evaluate the integral by Cauchy integral formula

$$\int \frac{4-3z}{z(z-1)(z-2)} dz \text{ where C is the circle } |Z| = \frac{3}{2}.$$

SECTION – C

6. (a) Evaluate $\int_0^\infty \frac{x^2}{x^6+1} dx$ using complex integration.

   (b) Expand $\frac{1-\cos z}{z^3}$ about $z = 0$.

7. (a) The contents of Urn I, II and III are as follows 1 white, 2 black and 3 red balls, 2 white, 1 black and 1 red ball, and 4 white, 5 black and 3 red balls. One Urn is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from I, II or III ?

(b) Fit a Poisson distribution to the following data and calculate the theoretical frequencies :

x :	0	1	2	3	4	($e^{-0.97} = 0.379$)
f :	46	38	22	9	1	

SECTION – D

8. (a) In a normal distribution 31% of the item are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

(b) A sample of 20 items has mean 42 and standard deviation 5 units. Test the hypothesis that is a random sample from a normal population with mean 45 units.

9. Using Simplex Method solve the following L.P.P.

Maximize : $z = 2x_1 + 5x_2$

Subject to : $x_1 + 4x_2 \leq 24$

$3x_1 + x_2 \leq 21$

$x_1 + x_2 \leq 9$

$x_1, x_2 \geq 0$