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24022

B.Tech 3rd Semester AEIE (F-Scheme) Examination,

December–2014

MATHEMATICS–III

Paper–Math–201–F

Time allowed : 3 hours] *[Maximum marks : 100*

Note : ***Question No. 1 is compulsory.*** *Attempt total* ***five*** *questions with selecting* ***one*** *question from each section. All questions carry equal marks.*

1. (a) What are the Dirichlet conditions for the expansion of Fourier series in $[c, c + 2\pi]$?

(b) Express $f(x) = x$ as a half range sine series in $0 \leq x \leq 2$.

(c) Define an analytic function. State the necessary and sufficient conditions for a function to be analytic.

(d) Find the finite Fourier sine transform of $f(x) = 2x, 0 < |x| < 4$.

(e) Evaluate $\oint_C \frac{e^{-z}}{z+1} dz$, where C is the circle $|z| = \frac{1}{2}$

(f) Define degenerate and non-degenerate solution.

(g) If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$. Find $P(B/A)$

(h) What is the chance that a leap year should have fifty three Mondays ?

Section–A

2. (a) Explain $f(x) = \left(\frac{\pi - x}{2}\right)^2$, $0 < x < 2\pi$, as Fourier series Hence show that

(i) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots\ldots = \frac{\pi^2}{12}$

(ii) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots = \frac{\pi^2}{6}$

(b) Obtain a half range cosine series for

$$f(x) = \begin{cases} x, & \text{for } 0 \le x < \frac{\ell}{2} \\ (\ell - x), & \text{for } \frac{\ell}{2} \le x \le \ell \end{cases}$$

3. (a) Find the Fourier Sine transform of $\frac{1}{x(x^2 + a^2)}$

(b) Express the function $f(x) = \begin{cases} 1 & \text{For } |x| \le 1 \\ 0 & \text{For } |x| > 1 \end{cases}$ as Fourier integral.

Hence Evaluate $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$

Section–B

4. (a) Prove that

$$\tan^{-1}(e^{i\theta}) = \left(\frac{n\pi}{2} + \frac{\pi}{4}\right) - \frac{i}{2} \log \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

(b) If $f(z)$ is a holomorphic function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$$

5. (a) Determine the analytic function whose imaginary part is $e^{-x}[x\cos y + y\sin y]$.

(b) Evaluate $\oint \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}\,dz$ where C is the circle $|z| = 3$.

Section–C

6. (a) Find the Laurent's series expansion of $\frac{z^2-2}{z^2+5z+6}$ in the region

(i) $2 < |z| < 3$ (ii) $|z| > 3$.

(b) Evaluate $\int_0^\infty \frac{x^2}{x^6+1}\,dx$ using complex integration.

7. (a) In a bolt factory, there are 4 machines A, B, C, D manufacturing 20%, 15%, 25% and 40% of the total output respectively. Of their output 5%, 4%, 3% and 2% in the same order, are defective bolts. A bolt is chosen at random from the Factory's production and is found defective. What is the probability that the bolt was manufactured by machine A or machine D ?

(b) Let x be a random variable defined by the density function

$$f(x) = \begin{cases} 3x^2, 0 \le x \le 1 \\ 0, \text{ otherwise} \end{cases}$$

Find $E(x)$, $E(3x - 2)$, $E(x^2)$.

Section–D

8. (a) In a hospital 475 female and 525 male babies were born in a week. Do these figure confirm the hypothesis that males and females are born in equal number.

(b) A set of 5 coins is tossed 3200 times and the number of heads appearing each time is noted. The results are given below

No of heads :	0	1	2	3	4	5
Frequency :	80	570	1100	900	500	50

Test the hypothesis that coins are unbiased.

9. Use the Simplex method to solve the following L.P.P.

Maximize $Z = x_1 - 3x_2 + 2x_3$

Subject to

$$3x_1 - x_2 + 2x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0$$