B.Tech. 3rd Semester AEIE F Scheme Examination,
December–2016

MATHEMATICS–III

Paper–Math–201–F

Time allowed : 3 hours]   *[Maximum marks : 100*

*Note : Question No. 1 is compulsory. Attempt five
questions by taking one from each unit. All
questions carry equal marks.*

1. (a) Expand $(\pi x - x^2)$ in a half range sine series in the
   interval $(0, \pi)$ up to first three terms.   5

   (b) Separate $\log (\sin z)$ into real and imaginary parts
   where $z = x + iy$.   5

   (c) Expand " $\dfrac{1}{z^4 + 3z^2 + 2}$ " as Laurent's series
   valid for $1 < |z| < \sqrt{2}$.   5

   (d) Solve the following LPP by graphical
   method

   Minimize $z = 20x_1 + 10x_2$

   subject to constrains $x_1 + 2x_2 \le 40$; $4x_1 + 3x_2 \ge 60$

        $3x_1 + x_2 \ge 30$; $x_1, x_2 \ge 0$ 5

Unit–I

2. (a) If $f(x) = \begin{cases} 0 & ; \text{ for } -\pi \le x \le 0 \\ \sin x & ; \text{ for } 0 \le x \le \pi, \end{cases}$ then prove

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin x - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2nx}{4n^2 - 1}$$ 10

(b) Find the Fourier transform of

$$f(x) = \begin{cases} 1-x^2 ; & \text{if } |x| < 1 \\ 0 & ; \text{if } |x| > 1 \end{cases}$$

and use it to evaluate

$$\int_{0}^{\infty}\left(\frac{x\cos x - \sin x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx.$$ 10

3. (a) Find Fourier sine transform of $\dfrac{1}{(x^3 + xa^2)}.$ 10

(b) Find Fourier sine and cosine series of

$$f(x) = \begin{cases} x; & 0 < x < \dfrac{\pi}{2} \\ 0; & \dfrac{\pi}{2} < x < \pi \end{cases}$$ 10

Unit–II

4. (a) Determine the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$ 8

(b) By using Cauchy's integral formula evaluate

(i) $\oint_C \left(\dfrac{e^z}{z^2 + 2z + 1} \right)^2 dz$, where C is the circle

$|z| = 2$ 6

(ii) $\oint_C \dfrac{\cos \pi z^2}{z^2 - 3z + 2} dz$, where C is the circle

$|z| = 2.5$ 6

5. (a) If $\tan(\theta + i\phi) = \cos \alpha + i \sin \alpha$, then prove that

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4} \text{ and } \phi = \frac{1}{2} \log \tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \qquad 10$$

(b) State and prove Cauchy's Integral formula.   10

Unit–III

6. (a) Evaluate $\oint_C \dfrac{z^2 - 3z}{z^3 + 2z^2 + 5z} dz$, where C is the circle (i) $|z + 1 - i| = 2$, (ii) $|z + 1 + i| = 2$

$$6+6=12$$

(b) Prove that $\displaystyle\int_0^{2\pi} \dfrac{\sin^2 \theta}{a + b \cos \theta} d\theta + \dfrac{2\pi}{b^2} (a - \sqrt{a^2 - b^2})$

where $0 < b < a$.   8

7. (a) In a bolt factory there are four machines A, B, C, D manufacturing 20%, 15%, 25%, and 40% of total output respectively. Of their output 5%, 4%, 3% and 2% in the same order are defective bolts. A bolt is chosen at random from the factory's production and is found defective. What is the probability that the bolt was manufactured by machine 'A' or machine 'D'?   10

(b) Fit the normal curve to the following distribution

x :	2	4	6	8	10
f :	1	4	6	4	1

Unit–IV

8. Using Simplex method maximize, $z = 3x_1 + 5x_2 + 4x_3$ subject to $2x_1 + 3x_2 \leq 8$; $2x_2 + 5x_3 \leq 10$

$$3x_1 + 2x_2 + 4x_3 \leq 15 ; x_1, x_2, x_3 \geq 0 \qquad 20$$

9. (a) Verify whether Poisson distribution can be assumed from the data given below :   10

No. of defects	0	1	2	3	4	5
Frequency	6	13	13	8	4	3

(b) The nine items of a sample have the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these values differ significantly from the assumed mean 47.5.   10