Roll No. .....

24022

B. Tech. 3rd Semester (CSE) Examination – December, 2018

MATHEMATICS - III

Paper : Math-201-F

Time : Three Hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, by selecting ***one*** Question from each Section. Question No. **1** is ***compulsory.***

1. ***Compulsory*** question :

(a) State Dirichlet's condition for existence of fourier series of a function.

(b) If $F_s(S)$ and $F_c(S)$ are fourier sine and cosine transform of $f(x)$ respectively, then show that :

(i) $F_s(xf(x)) = \frac{-d}{dx}\{F_c(s)\}$

(ii) $F_c(xf(x)) = \frac{d}{ds}\{F_s(s)\}$

(c) Show that :

$$f(z) = \frac{x^2y^3(x+iy)}{x^6+y^{10}},\ z \neq 0,\ f(0) = 0$$

is not analytic at origin.

(d) Evaluate :

$$\int_0^{1+i} (x^2 - iy)\, dz \text{ along the path } y = x^2$$

(e) Find the radius of convergence of the power series :

$$\sum \frac{2^{-n} z^n}{1+in^2}$$

(f) A variate X has the probability distribution :

X :	–3	6	9
P[X = x] :	1/6	1/2	1/3

Find E $\left[(2x+1)^2\right]$.

(g) Write a short note on :

(i) Test of significance

(ii) Errors

(h) Solve graphically Max $z = 3x + 4y$, subject to the constraints $2x + 4y \leq 40;\ 2x + 5y \leq 180, x \geq 0, y \geq 0$.

20

SECTION – A

2. (a) Find the fourier series expansion of $f(x)$ if : 10

$$f(x) = \begin{cases} -\pi & , \quad -\pi < x < 0 \\ x & \quad 0 < x < \pi \end{cases}$$

and hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$.

(b) Develop $f(x)$ in a fourier series in the interval (0,2) if. : 10

$$f(x) = x, 0 < x < 1$$

$$= 0, 1 < x < 2$$

3. (a) Express $f(x)=\begin{cases}1 & , \quad 0 \le x \le \pi \\ 0 & \quad x > \pi\end{cases}$ as a fourier sine integral and hence evaluate

$$\int_0^\infty \frac{1-\cos(\pi\lambda)}{\lambda} \sin(x\lambda)\, d\lambda .$$ 10

(b) Find the fourier cosine transform of e^{-x^2}. 10

SECTION – B

4. (a) Express Log (log i) in the form $A+iB$. 10

(b) If $\tan(\theta+i\phi)=\tan\alpha+i\sec\alpha$, show that : 10

$$e^{2\phi}=\pm\cot\alpha/2 \text{ and } 2\theta=\left(n+\frac{1}{2}\right)\pi+\alpha$$ 10

5. (a) If $w=\phi+i\psi$ represents the complex potential function for an electric field and $\psi=x^2-y^2+\frac{x}{x^2+y^2}$, determine ϕ. 10

(b) Evaluate $\oint_C \frac{e^{-2z}}{(z+1)^3} dz$, where C is the circle $|z| = 2$ by using Cauchy's integral formula. 10

SECTION – C

6. (a) Expand $\frac{e^{2z}}{(z-1)^3}$ about the singularity $z = 1$ in Laurent series. 10

(b) Evaluate the integral

$$\int_0^{2\pi} \frac{d\theta}{1 - 2r\sin\theta + r^2}, (0 < r < 1).$$ 10

7. (a) A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y. 10

(b) A certain screw making machine produces on average 2 defective screw out of 100, and packs them in boxes of 500. Find the probability that a box contains 15 defective screws. 10

SECTION – D

8. (a) 325 men out of 600 men chosen from a big city were found to be smokers. Does this information support the conclusion that the majority of men in the city are smokers. 10

(b) Two independent samples of size 7 and 9 have the following values: 10

Sample A :	10	12	10	13	14	11	10	–	–
Sample B	10	13	15	12	10	14	11	12	11

Test whether the difference between the mean is significant.

9. (a) Using Simplex method, solve : 10

Max $z = x_1 + x_2 + 3x_3$

Subject to $3x_1 + 2x_2 + x_3 \leq 3, 2x_1 + x_2 + 2x_3 \leq 2,$ $x_1, x_2, x_3 \geq 0$.

(b) Using Dual simplex method, solve : 10

Min $z = x_1 + 2x_2 + 3x_3$

Subject to :

$2x_1 - x_2 + x_3 \geq 4, x_1 + x_2 + 2x_3 \leq 8, x_2 - x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$.
