3057

B. Tech 3rd Semester (ME)
Examination – December, 2019

MATHEMATICS – III
(PDE, Probability & Statistics)
Paper : BSC-ME-203-G

Time : Three Hours]   *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Question No. **1** mandatory. Attempt *one* question from each Unit.

1. (a) Define Partial Differential Equation with example.

$$6 \times 2.5 = 15$$

    (b) Define Non-homogeneous Partial Differential Equation with example.

(c) Define Poisson Distribution and mention its Assumptions.

(d) Define Random variable and mention its types.

(e) What do you mean by Normal Distribution ? Mention its properties.

(f) A Random variable X has the following probability function :

X :	0	1	2	3	4	5	6	7
p(X) :	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find the value of k.

UNIT – I

2. (a) Solve Lagrange's linear partial differential equation : 7.5

$$x(y^2 - z^2) p + y (z^2 - x^2) q = z(x^2 - y^2)$$

(b) Solve : 7.5

$$(D^2 - 2DD' + D'^2) z = e^{x+2y}$$

3. (a) Solve : 7.5

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = \cos(x + 2y)$$

(b) Obtain D' Alembert's solution of wave equation :

$$\frac{\partial^2 u}{dx^2} = \left(\frac{1}{c^2}\right)\left(\frac{\partial^2 u}{\partial t^2}\right)$$

give that initial deflection u(x,0) = f(x) and initial velocity : 7.5

$$\left(\frac{\partial u}{\partial t}\right)_{t=0} = g(x)$$

UNIT – II

4. (a) Solve the following partial differential equation by method of separation of variables :

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \text{ where } u(x, 0) = 6e^{-3x}. \qquad 7.5$$

(b) Solve one-dimensional heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{with} \quad \text{boundary} \quad \text{conditions}$$

$u(x, 0) = 6 \sin n \pi x, u(0, t) = 0$ and $u(1, t) = 0$.

7.5

5. Obtain solution of Laplace's equation in cylindrical polar co-ordinates using method of separation of variables. 15

UNIT – III

6. (a) From an urn containing 3 red and 2 white balls. A man is to draw 2 balls at random without replacement, being promised Rs. 20 for each red ball he draws and Rs. 10 for each white one. Find his expectation. 7.5

(b) State and prove Chebyshev's inequality. 7.5

7. (a) (i) Define discrete random variable and mention its properties. 2.5

(ii) A random variable gives measurements X between 0 and 1 with a probability function

$$f(x) = 12x^3 - 21x^2 + 10x; \quad 0 \le x \le 1$$

$$= 0$$

Find $P\left(X \le \frac{1}{2}\right)$ and $P\left(X > \frac{1}{2}\right).$ 5

(b) In a bolt factory, there are four machines A, B, C, D manufacturing 20%, 15%, 25% and 40% of total output respectively of their outputs 5%, 4%, 3% and 2% in the same order are defective. A bolt is chosen at random and found to be defective. What is the probability that the bolt was manufactured by machine A ? 7.5

UNIT – IV

8. (a) The mean height of 500 students is 151 cm and the Standard Deviation is 15cm. Assuming that the heights are normally distributed. Find how many students heights lie between 120 and 155cm. 7.5

(b) The average income of persons was Rs. 210 with a Standard Deviation of Rs. 10 in a sample of 100 people of a city. For another random sample of 150 persons the average income was Rs. 220 with Standard Deviation of Rs. 12. The Standard Deviation of income of people of the city was Rs. 11. Test whether there is any significant difference between the average incomes of the localities. 7.5

9. (a) Find the Rank correlation for the following data : 7.5

x :	56	42	72	36	63	47	55	49	38	42	68	60
y :	147	125	160	118	149	128	150	145	115	140	152	155

(b) The sales in a Supermarket during a week are given below test the hypothesis that the sales do not depend on the day of the week, using a significant level of 0.05. 7.5

Days :	Mon	Tue	Wed	Thu	Fri	Sat
Sales : (in '1000' Rs.)	65	54	60	56	71	84

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