

Roll No.

24022

B. Tech. 4th Sem. (E. E./

E.E.E./E.C.E./E.I.E./ I.

C.E./I.T./M.E./C.S.E./B.M.E./Civil Engg.)

(Common to All These Branches)

Examination – May, 2011

MATHEMATICS

Paper : Math-201-F

Time : Three hours]

[Maximum Marks : 100

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all. Question No. 1 is *compulsory*. Attempt *four* questions selecting *one* from each *Section (A to D)*. All questions carry equal marks.

1. (a) Express $f(x) = x$ as a Fourier series in the interval $-\pi < x < \pi$.
- (b) Find the Fourier transforms of $f(u) = \begin{cases} 1, & |u| < u_0 \\ 0, & |u| > u_0 \end{cases}$.
- (c) If the potential function is $\log(x^2 + y^2)$, find the flux function and the complex potential function.

- (d) In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails.
- (e) Using graphical method, solve the following L.P.P.

Maximize $Z = 2x_1 + 3x_2$ Subject to

$$x_1 - x_2 \leq 2$$

$$x_1 + x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

SECTION - A

2. (a) Find the Fourier expansion for the function :

$$f(x) = e^x \text{ in } -\pi < x < \pi.$$

- (b) Obtain the Fourier series expansion for the function $f(x) = x^2$ in $[-\pi, \pi]$ and deduce the following from it :

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

3. (a) Find the Fourier cosine transform of $f(x) = \frac{1}{(1+x^2)}$.

- (b) Verify convolution theorem for $f(x) = g(x) = e^{-x^2}$.

SECTION - B

4. (a) If $\tan(\theta + i\phi) = e^{i\alpha}$ show that $\theta = \left(n + \frac{1}{2}\right) \frac{\pi}{2}$ and

$$\phi = \frac{1}{2} \log \tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right).$$

(b) If $\sin^{-1}(x + iy) = \log(A + iB)$, show that

$$\frac{x^2}{\sin^2 u} - \frac{y^2}{\cos^2 u} = 1, \text{ where } (A^2 + B^2) = e^{2u}$$

5. (a) Prove that the function $f(z)$ defined by :

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} \quad (z \neq 0), \quad f(0) = 0$$

is continuous and the Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist.

(b) Evaluate $\int_C \frac{e^z}{(z^2 + \pi^2)^2} dz$, where C is $|z| = 4$.

SECTION - C

6. (a) Find Taylor's expansion of $f(z) = \frac{2z^3 + 1}{z^2 + z}$ about the point $z = i$.

(b) What type of singularity have the following functions :

$$(i) \frac{e^{2z}}{(z-1)^4} \quad (ii) \quad z e^{\frac{1}{z^2}} \quad (iii) \quad \frac{1}{1 - e^z}.$$

7. (a) In a lottery, m tickets are drawn at a time out of n tickets numbered from 1 to n . Find the expected values of the sum of the numbers on the tickets drawn.

(b) Fit a normal curve to the following distribution :

$x:$ 2 4 6 8 10

$f:$ 1 4 6 4 1

SECTION – D

8. (a) A group of 10 rats fed on a diet A and another group of 8 rats fed on a different diet B, recorded the following increase in weights :

Diet A: 5 6 8 1 12 4 3 9 6 10 gm

Diet B: 2 3 6 8 10 1 2 8 gm

Does it show the superiority of diet A over that of B.

(b) The means of simple sample of sizes 1,000 and 2,000 are 67.5 and 68.0 cm respectively. Can the samples be regarded as drawn from the same population of S. D. 2.5 m.

9. (a) Using dual simplex method, solve the following problem :

minimize $Z = 2x_1 + 2x_2 + 4x_3$, subject to

$$2x_1 + 3x_2 + 5x_3 \geq 2, 3x_1 + x_2 + 7x_3 \leq 3;$$

$$x_1 + 4x_2 + 6x_3 \leq 5, x_1, x_2, x_3 \geq 0$$

(b) Solve the following L.P.P. by simplex method :

minimize $Z = x_1 - 3x_2 + 3x_3$, subject to

$$3x_1 - x_2 + 2x_3 \leq 7; 2x_1 + 4x_2 \geq -12, -4x_1 + 3x_2$$

$$+8x_3 \leq 10, x_1, x_2, x_3 \geq 0$$