*Roll No.* ..................

# 24022

## B. Tech. 4th Semester

## Examination – May, 2012

### MATHEMATICS

**Paper : Math-201-F**

**(Common Paper)**

*Time : Three Hours ]* *[ Maximum Marks : 100*

*Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.*

*Note :* Attempt *five* questions in total, selecting one question from each Unit. Question **1** is ***compulsory***. All questions carry equal marks.

**1.** (a) Show that the series :

$\frac{4}{\pi}\left[\sin\frac{\pi x}{l} + \frac{l}{3}\sin\frac{3\pi x}{l} + ........\right]$ is equal to $l$ when $0 < x < l$.

(b) Show that $\int_0^\infty \frac{\cos\lambda}{\lambda^2 + 1}\, d\lambda = \frac{\pi}{2} e^{-x}, x \geq 0$.

(c) If $f(z) = u + iv$ is an analytic function of $z$, find $f(z)$ if $u - v = [x - y)(x^2 + 4xy + y^2)$.

(d) Show that for every path between the limit :

$$\int_{-2}^{-2+i} (2+z)^2\, dz = \frac{-i}{3}$$

(e) Evaluate $\int_C \frac{z^2 - z + 1}{z-1}\, dz$, where $C$ is the circle $|z| = 1$.

(f) If the probability that a new born child is male is 0.6. Find the probability that is a family of 5 children, there are exactly 3 boys.

(g) The average marks in English of a sample of 100 is 51 with a S.D. of 6 marks could this have a random sample from a population with average marks 50.

(h) Intelligent tests given of two groups of boys and girls :

| | Mean | S.D | Size |
|---|---|---|---|
| Girls | 75 | 8 | 60 |
| Boys | 73 | 10 | 100 |

Examine if the difference between mean scores is significant.

## UNIT – A

**2.** (a) Find a series of cosines of multiples of x which will represent $\log\left(2sm\frac{1}{2}x\right)$ in the interval $[0, \pi]$.

(b) Obtain Fourier series for the function :

$$f(x) = \begin{cases} \pi x & , 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$$

3. (a) Find the sine and cosine transforms of :

$$\frac{e^{ax} + e^{-ax}}{e^{\pi x} - e^{-\pi x}}$$

(b) State and prove convolution theorem for Fourier transforms.

## UNIT – B

4. (a) Separate into real and imaginary parts $\sin^{-1}(\cos\theta + i\sin\theta), \quad 0 < \theta < \frac{\pi}{2}$.

(b) Prove that the function $\sin z$ is analytic and find its derivative.

5. (a) If $f(z)$ is an analytic function with constant modulus, show that f(z) is constant.

(b) If $f(z) = \oint_C \frac{4z^2 + z + s}{z - \xi} dz$, where C is the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, find $f''(-i)$.

## UNIT – C

6. (a) Find the series expansion of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ about $z = 0$ in the region :

(i) $|z| < 2$ (ii) $2 < |z| < 3$

(b) Prove that :

$$\int_0^{2\pi} \frac{\sin^2\theta}{a + b\cos\theta} d\theta = \frac{2\pi}{b^2}\left[a - \sqrt{a^2 - b^2}\right], \text{ where } 0 < b < a$$

7. (a) If the variance of the Poisson distribution is 2. Find the probabilities for r = 1, 2, 3, 4 from the recurrence relation of the Poisson distribution.

(b) Find the mean and variance of normal contribution.

## UNIT – D

8. (a) Using simplex method :

Minimize $z = 5x_1 + 3x_2$ subject to $x_1 + x_2 \le 2$, $5x_1 + 2x_2 \le 10$, $3x_1 + 8x_2 \le 12$, $x_1, x_2 \ge 0$.

(b) Solve the dual problem of the following LPP :

Minimize $z = 20x_1 + 30x_2$, subject to

$3x_1 + 3x_2 \le 36$, $5x_1 + 2x_2 \le 50$, $2x_1 + 6x_2 \le 60$, $x_1, x_2 \ge 0$.

9. (a) A sample of 20 items has mean 42 units and S.D. 5 units. Test the hypothesis that it is a random. sample from a normal population with mean 45 units.

(b) Sample of sizes 10 and 14 were taken from two normal populations with S.D. 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. Test wheater the means of two populations are same at 5% level.