Roll No. .....

24022

B.Tech 4th Semester (E.E.) Examination – May, 2013

MATHEMATICS – III

Paper : Math-201-F

Time : Three hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : ***Question 1st is compulsory***. Attempt total five questions with selecting ***one*** question from each Section. All questions carry equal marks.

1. (a) If X is a normal variable with mean 30 & standard deviation 5 : find the probabilities that

(a) $20 \le X \le 40$

(b) $X \le 45$

(b) Prove that $\sin(\alpha + n\theta) - e^{i\alpha} \sin n\theta = e^{-in\theta} \sin \alpha$

(c) State Bayes theorem.

(d) Separate into real and imaginary part of $\tan(x+iy)$

(e) Find the finite Fourier sine and cosine transform of $f(x) = 2x. 0 < |x| < 4$

(f) Define slack and surplus variables.

(g) Find the value of b_n in the Fourier series of $f(x) = |x|$ in $(-\pi, \pi)$

(h) Express the function $f(x)$ as a Fourier integral,

$$f(x) = \begin{cases} 1 \text{ for } |x| < 1 \\ 0 \text{ for } |x| > 1 \end{cases}$$

SECTION – A

2. (a) Expand $f(x) = x \sin x, 0 < x < 2\pi$, in a Fourier series.

(b) Expand $f(x)$ as a Fourier series if :

$$f(x) = \pi x, \ 0 < x < 1$$
$$= \pi(2 - x), \ 1 < x < 2$$

3. (a) Find the Fourier sine transform of $\frac{1}{x(x^2 + a^2)}$

(b) Find the Fourier Transform of

$$f(x) = \begin{cases} 1 - x^2 \text{ if } |x| < 1 \\ 0 \quad \text{if } |x| > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$

SECTION – B

4. (a) If $\tan(\theta + i\varphi) = \tan\alpha + i\,\mathrm{Sec}\,\alpha$, show that

(i) $e^{2\varphi} = \pm\cot\alpha/2$

(ii) $2\theta = (n + 1/2)\pi + \alpha$

(b) Determine the analytic function whose real part is $e^{2x}(xCos2y - y\sin 2y)$

5. (a) Define line integral of $f(z)$. Prove that

$\int_C \frac{dz}{z} = -\pi i \text{ or } \pi i$, according as c is the semi-circular arc $|z| = 1$ above or below of the real axis.

(b) Using Cauchy's Integral Formula, Evaluate :

$\oint \frac{\sin^6 z}{(z - \pi/3)^3} dz$ around the circle $|z| = 1$

SECTION – C

6. (a) Expand $e^{2z}/(z-1)^3$ about the singularity z = 1 in Laurent's series.

(b) Evaluate $\int_0^{\pi} \frac{1}{a + b\cos\theta} d\theta$, where a > b

7. (a) In a bolt factory, there are 4 machines A, B, C, D manufacturing 20%, 15%, 25% and 40% of the total output respectively. Of their outputs 5%, 4%, 3% and 2% in the same order, are defective bolts. A bolt is chosen at random from the factory's production and is found defective. What is the

probability that the bolt was manufactured by machine A or machine D ?

(b) Fit a Normal curve to the following distributions :

x :	2	4	6	8	10
f :	1	4	6	4	1

8. A survey of 320 families with 5 children each revelled the following distribution :

No of boys :	5	4	3	2	1	0
No of girls :	0	1	2	3	4	5
No of families :	14	56	110	88	40	12

Is this result consistent with the hypothesis that male and female births are equally probable ?

9. Solve the following L.P.P. by simplex method.

Minimize $z = x_1 - 3x_2 + 3x_3$

Subject to

$$3x_1 - x_2 + 2x_3 \leq 7,$$
$$2x_1 + 4x_2 \geq -12,$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10;$$
$$x_1, x_2, x_3 \geq 0$$