24022

[Graph Paper]

B.Tech. 4th Semester (BME) F Scheme

Examination, May–2014

MATHEMATICS-III

Paper–MATH-201-F

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt **five** questions in all. **Question No.1 is compulsory.** Attempt **four** questions selecting **one** question from each section **(A-D)**. All questions carry equal marks.*

1. (a) Draw and write the name of the wave form for the following function :

$$f(x) = \begin{cases} a & \text{for } 0 < x < \pi \\ 0 & \text{for } \pi < x < 2\pi \end{cases}$$

(b) Define Dirichlet's conditions for a Fourier series.

(c) Find the Fourier Transform of

$$f(x) = \begin{cases} 1 & 0 < x < a \\ 0 & x > a \end{cases}$$

(d) Separate tan (x + iy) into real and imaginary

(f) Find the residue at $z = 0$ for $z \cos\left(\frac{1}{z}\right)$

(g) Find the probability of getting 4 heads and 6 tosses of a fair coin.

(h) The heights of college students in a city are normally distributed with S.D. 6 cms. A sample of 1000 students has mean height 158 cms. Test the hypothesis that the mean of college students in the city is 160 cms.

Section–A

2. (a) Determine the Fourier Series expansion of the function $f(x) = \pi^2 - x^2$ for $-\pi \leq x \leq \pi$. Also deduce the Fourier series of $f(x) = x$ in $[-\pi, \pi]$ from the Fourier series of $f(x) = \pi^2 - x^2$.

(b) Find the Fourier Series expansion of $f(x) = x.\cos\left(\frac{\pi x}{L}\right)$ in the interval $-L \leq x \leq L$.

3. (a) State and prove Convolution Theorem of Fourier Transform.

(b) Find the Fourier Transform of

$$\begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

Section–B

4. (a) If $\sin(\theta + i\varphi) = \cos\alpha + i \sin\alpha$, prove that $\cos^4\theta = \sin^2\alpha = \sin h^4\varphi$.

   (b) Find the values of A and B such that the function $f(z) = x^2 + Ay^2 - 2xy + i(Bx^2 - y^2 + 2xy)$ is analytic. Also find $f'(z)$.

5. (a) If $w = \phi + i\psi$ represents the complex potential for an electric field and

   $$\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$$ determine the function ϕ.

   (b) Verify Cauchy's Integral Theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $1 + i$, $-1 + i$ and $-1 - i$.

Section–C

6. (a) Find the Laurent's series expansion valid in the region $0 < |z - 1| < 1$ for the function

   $$f(z) = \frac{2z+1}{z^3 + z^2 - 2z}$$

(b) z^2

7. (a) If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals
   (i) exactly 3
   (ii) more than 2 individuals
   (iii) none
   (iv) more than one individual will suffer a bad reaction.

   (b) Fit a normal curve to the following distribution :

x	2	4	6	8	10
f	1	4	6	4	1

Section–D

8. (a) A sample of 6 persons in an office revealed an average daily smoking of 10, 12, 8, 9, 16, 5 cigarettes. The average level of smoking in the whole office has to be estimated at 90% level of confidence. $t = 2.015$ for 5 degree of freedom.

   (b) Fit a Poisson distribution to the following data and test the goodness of test

x	0	1	2	3	4
f	46	38	22	9	1

9. Using Graphical and Dual Simplex Method to solve the following Linear Programming Problem;