

(4)

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9. Maximize :  $Z = -3x_1 - 2x_2$

subject to :

$$x_1 + x_2 \geq 1,$$

$$x_1 + x_2 \leq 7,$$

$$x_1 + 2x_2 \geq 10,$$

$$x_2 \leq 3,$$

$$x_1, x_2 \geq 0$$

by using dual simplex method.

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(Graph Paper)

B. Tech. 4th Semester (BME) Examination,

May-2016

MATHEMATICS-III

Paper-Math-201-F

Time allowed : 3 hours]

[Maximum marks : 100

**Note :** Attempt five questions in total, selecting at least one question from each section. Q. No. 1 is compulsory.

1. (a) Find  $a_n$  for  $f(x) = x - x^2$ ,  $-1 < x < 1$ .
- (b) Examine  $\sin z$  is analytic or not analytic.
- (c) Define residue at a pole and state Cauchy's residue theorem.
- (d) Minimize :  $Z = 20x_1 + 10x_2$   
subject to :

$$x_1 + 2x_2 \leq 40,$$

$$3x_1 + x_2 \geq 30,$$

$$4x_1 + 3x_2 \geq 60,$$

$$x_1, x_2 \geq 0$$

by graphical method.

Section-A

2. (a) Find Fourier series for

$$f(x) = \begin{cases} \pi + x & \text{for } -\pi < x < 0 \\ 0 & \text{for } 0 \leq x < \pi \end{cases}$$

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[Encl. Graph Paper

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[P.T.O.]

- (b) Obtain half range cosine series for

$$f(x) = x, \quad 0 < x < \frac{\pi}{2},$$

$$= \pi - x, \quad \frac{\pi}{2} < x < \pi.$$

3. Solve:

$$\frac{\partial V}{\partial t} = k \frac{\partial^2 V}{\partial x^2} \text{ for } x > 0, t > 0 \text{ under the boundary}$$

conditions  $V = V_0$  when  $x = 0, t > 0$  and the initial condition  $V = 0$  when  $t = 0, x > 0$ .

#### Section-B

4. (a) If  $\tan(\theta + i\phi) = \cos \alpha + i \sin \alpha = e^{i\alpha}$ ,  
prove that:

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4} \text{ and } \phi = \frac{1}{2} \log \tan \left( \frac{\pi}{4} + \frac{\alpha}{2} \right).$$

- (b) Show that the function  $f(z) = \sqrt{|xy|}$  is not analytic at the origin, even though C. R. equations are satisfied there at.

5. (a) State and prove Cauchy's integral theorem.

- (b) Evaluate:

$$\oint_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz \text{ where } C \text{ is the circle } |z| = 3.$$

#### Section-C

6. (a) Find the series expansion of

$$f(z) = \frac{z^2 - 1}{z^2 + 5z + 6} \text{ about } z = 0 \text{ in the region}$$

$$(i) |z| < 2$$

$$(ii) 2 < |z| < 3.$$

- (b) Evaluate:

$$\oint_C \tan z \, dz, \text{ where } C \text{ is the circle } |z| = 2.$$

7. (a) Prove that the total area under normal probability curve is unity.
- (b) A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A were defective and 1% produced by machine B were defective. If a defective item is drawn at random, what is the probability that it was produced by machine A?

#### Section-D

8. Verify whether binomial distribution can be assumed from the data given below:

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4