

B.E.
Sixth Semester Examination, May-2009
Machine Design-II (ME-304-E)

Note : Attempt *ALL* five questions. All questions carry equal marks.

Q. 1. (i) Enumerate the various manufacturing methods of machine parts which a designer should know.

Ans. The various manufacturing methods (processes) which a designer should consider can be divided into two groups :

- (i) Process which are used for the preliminary shaping of the machine parts, mostly by the use of heat.
- (iii) Process which gives final shape of the machine parts, by cold machining.

Process included in the first group are casting, forging, riveting and welding. Each process has its possibilities and limitations.

Second group includes the following processes : turning, boring, planing, shaping, milling, drilling, reaming, broaching, spot facing, grinding, gear and screw cutting, tapping, honing and polishing.

Q. 1. (ii) Write a brief note on the design of castings?

Ans. It is one of the most widely used manufacturing methods in mechanical engineering. The following are the most commonly used processes for iron and steel castings (i) Sand casting (ii) Permanent mould casting, (iii) Shell moulding (iv) Centrifugal casting (v) Die casting.

The following basic rules should be followed in the design of casting :

- (i) The casting should be designed with uniform-thickness of possible. An abrupt change in section thickness must always be avoided to avoid shrinkage defects and stress concentration at sharp corners.
- (ii) The minimum thickness for gray iron casting can be as low as 3mm, but the average value is 6mm.
- (iii) Ribs should be used chiefly for static loads. These should be avoided where impact loads are expected since these increases the rigidity of the parts.
- (iv) The iron casting should be loaded in compression as far as possible, since cast iron is much more stronger in compression than in tension.
- (v) Don't use iron casting for impact and shock loading.
- (vi) Don't use cast iron at temperatures above 300°C since its strength decreases after 400°C.
- (vii) Avoid concentration of material so that no shrinkage cavities are formed.
- (viii) Parting lines should be made as even as possible.
- (ix) Pattern should be provided with ample draft for their easy withdrawal from the mould.
- (x) If possible, the casting should be so designed that no cores are required.

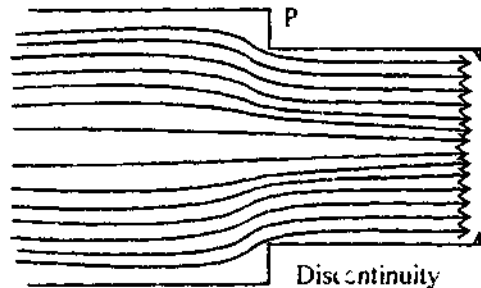
Q. 1. (iii) Define and differentiate between form stress factor and stress concentration factor.

Ans. The presence of abrupt change in the section of the machine parts the stresses in the machine member do not follow the elementary equations. Localized high stresses occurs and the fibres nearest to the abrupt change in section are affected most as shown in fig. where the material in the neighbourhood of points 'P' is stressed considerably higher than the average values. Such a phenomenon is known as "stress concentration." The measure of stress concentration is the "stress concentration factor" which is defined as,

$$K_t = \frac{f_t \max}{f_t \text{nom}} \text{ for normal stress}$$

$$K_{ts} = \frac{f_s \max}{f_s \text{nom}} \text{ for shear stress}$$

Where $f_t \text{nom}$ and $f_s \text{nom}$ are based on the elementary formulas.



The factor K_t is usually called the theoretical stress concentration factor.

It depends only on the size of discontinuity or on its geometric shape. Therefore it is commonly called as "form stress factor." The actual "stress concentration factor" is different from, "form stress factor" since it depends on material and type of loading also. The effect of actual "stress concentration factor" is less than the "form stress factor."

The "form stress factor" K_t and the actual "stress concentration factor" K are connected by the following relation,

$$K = 1 + q(K_t - 1)$$

Where q is known as "Index of sensitivity" of the material to abrupt change of section from the above equation.

$$q = \frac{K - 1}{K_t - 1}$$

Q. 1. (iv) What are the salient features used in design of forgings? Explain.

Ans. A forging process can be defined as the plastic deformation of a metal caused by the stresses imposed on it through hammering or squeezing operations. The forging process can be classified as below :

- (i) Smith forging or hand forging.
- (ii) Drop forging.
- (iii) Machine forging or upset forging.
- (iv) Press forging.

The following points should be kept in mind while designing a forging :

- (i) The direction of fibre flow lines should be kept in mind, so that full advantages are taken of them.
- (ii) Adequate draft should be provided to permit easy removal of the forgings from the dies.
- (iii) If possible, the parting line on a forging should be in one plane.

- (iv) Ample fillets and curvatures should be given to the forging.
- (v) The parting lines, if possible should be divide the forging about in half.
- (vi) Ribs should be low and wide.
- (vii) Pockets and recesses should be minimum to avoid increased die wear.
- (viii) Section should not be so thin as to restrict the flow of metal.

Q. 2. A shaft is supported by two bearings placed 1m apart. A 600 mm diameter pulley is mounted at a distance of 300mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25kN. Another pulley 400mm diameter is placed 200mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulley s is 180° and $\mu = 0.24$. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

Ans. Data given

$$AB = 1\text{m}, DC = 600\text{mm or } R_C = 300\text{mm} = 0.3\text{m}$$

$$AC = 300\text{mm} = 0.3\text{m}, T_1 = 2.25\text{kN} = 2250\text{N}, D_D = 400\text{mm}$$

$$R_D = 200\text{mm} = 0.2\text{m}, BD = 200\text{mm} = 0.2\text{m}, \theta = 180^\circ = \pi$$

$$\mu = 0.24, f_b = 63\text{N/mm}^2, f_s = 42\text{N/mm}^2$$

The space diagram is as shown in fig.

T_1 = Tension on the tight side of the belt on C pulley.

$$= 2250\text{N}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} = \frac{T_1}{T_2} = e^{0.24 \times \pi} = 2.127$$

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058\text{N}$$

\therefore Vertical load acting on the shaft at C

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308\text{N}$$

The vertical load diagram is as shown in figures.

The torque acting on the pulley is

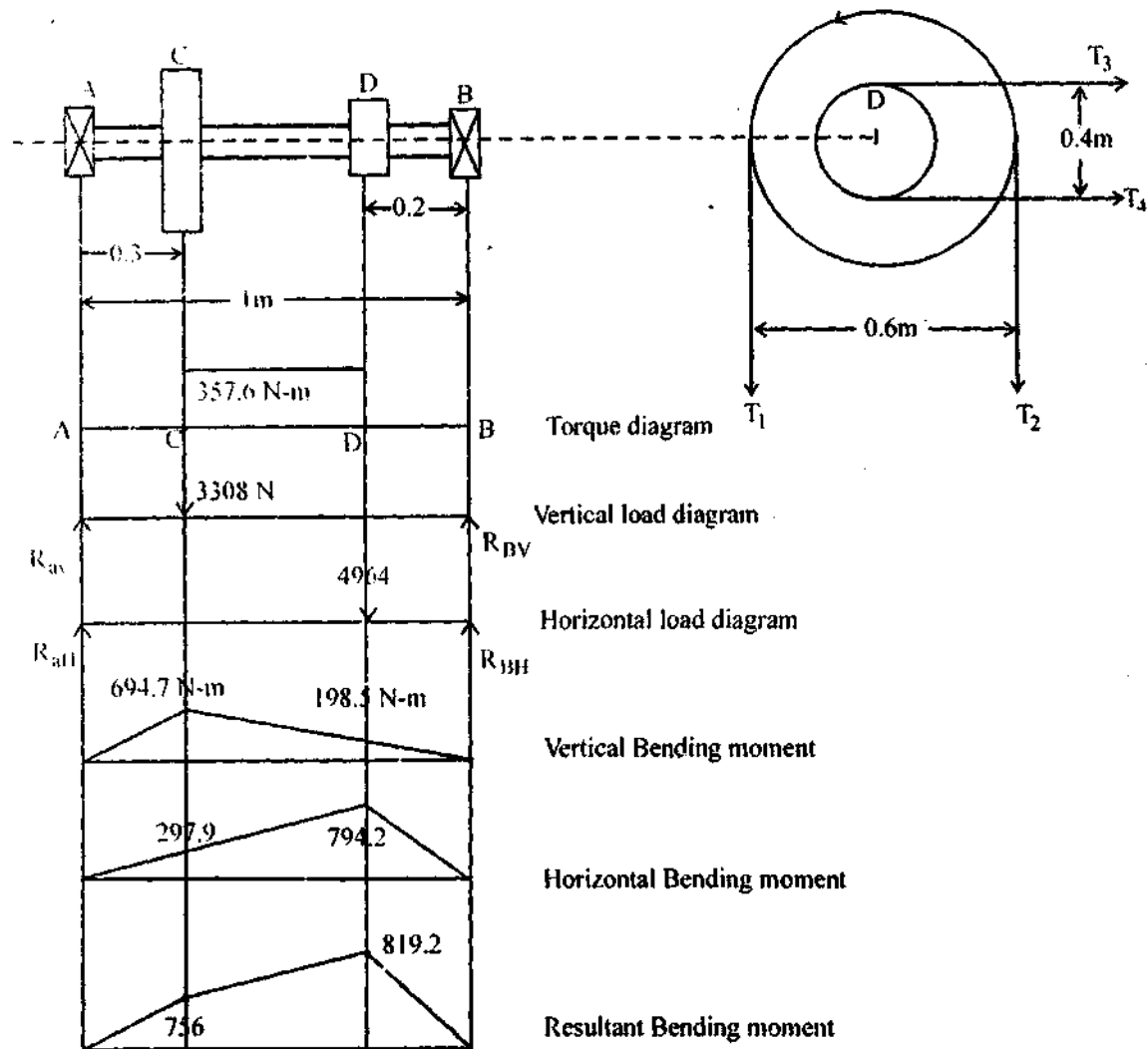
$$T = (T_1 - T_2)R_C$$

$$= (2250 - 1058) \times 0.3 = 357.6\text{N-m}$$

Since torque on the both pulley is same,

$$(T_3 - T_4) \times R_D = T = 357.6$$

Or
$$T_3 - T_4 = \frac{357.6}{0.2} = 1788\text{N}$$



Again θ is same so

$$\frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127$$

Or $T_3 = 2.127 T_4$

$$T_3 = 3376 \text{ \& } T_4 = 1588 \text{ N}$$

\therefore Horizontal load on the shaft at D

$$W_D = T_3 + T_4 = 3376 + 1588 = 4964 \text{ N}$$

The horizontal load diagram is as shown in fig.

Now find the maximum bending moment for vertical and horizontal loading.

First consider vertical loading at C

$$R_{AV} + R_{BV} = 3308\text{N}$$

Take moment about A.

$$R_{BV} \times 1 = 3308 \times 0.3 \text{ or } R_{BV} = 992.4\text{N}$$

$$R_{AV} = 3308 - 992.4 = 2315.6\text{N}$$

We know that BM at A & B is 0

$$\begin{aligned}\text{BM at C} \quad M_{CV} &= R_{AV} \times 0.3 = 2315.6 \times 0.3 \\ &= 694.7\text{ N-m}\end{aligned}$$

$$\text{BM at D} \quad M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5\text{ N-m}$$

The bending moment diagram is as shown in fig.

Now consider horizontal loading at D.

$$R_{AH} + R_{BH} = 4964 \times 1$$

Take moment about A

$$R_{BH} \times 1 = 4964 \times 0.8$$

or

$$R_{BH} = 3971\text{N}$$

$$R_{AH} = 4964 - 3971 = 993\text{N}$$

$$\text{BM at C} \quad M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9\text{ N-m}$$

$$\text{BM at D} \quad M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2\text{ N-m}$$

The bending moment diagram is as shown in fig. The resultant bending moment at C & D are

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756\text{ N-m}$$

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2\text{ N-m}$$

We see that bending moment of maximum at D

$$M = M_D = 819.2\text{ N-m}$$

Let d = diameter of the shaft

We know that equivalent twisting moment

$$\begin{aligned}T_e &= \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} \\ &= 894\text{ N-m} = 894 \times 10^3\text{ N-m}\end{aligned}$$

From the torsional equation,

$$894 \times 10^3 = \frac{\pi}{16} \times f_s \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25d^3$$

$$d^3 = \frac{894 \times 10^3}{8.25} = 108 \times 10^3 \text{ or } d = 47.6 \text{ mm}$$

Again for the equivalent bending moment equation,

$$M_e = \frac{1}{2} M + \sqrt{M^2 + T^2} = \frac{1}{2} (M + T_e)$$

$$= \frac{1}{2} (819.2 + 894) = 856.6 \text{ N-m}$$

$$= 856.6 \times 10^3 \text{ N-mm}$$

$$856 \times 10^3 = \frac{\pi}{32} \times f_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2d^3$$

$$d^3 = \frac{856 \times 10^3}{6.2} = 138.2 \times 10^3 \text{ or } d = 51.7 \text{ mm}$$

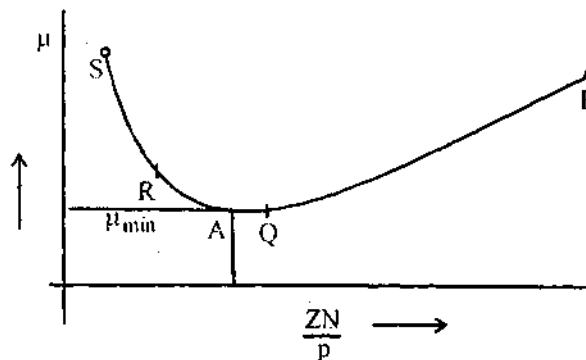
Taking larger of the two values we have $d = 51.7 \text{ mm}$

$d = 55 \text{ mm}$ from the standard values.

Ans.

Q. 3. (i) Define bearing number and Sommerfield number.

Ans. The coefficient of friction in design of bearing is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiment that the coefficient of friction for a full lubricant journal bearing is a function of three variables



(i) $\frac{ZN}{p}$

(ii) $\frac{d}{c}$

(iii) $\frac{1}{d}$

The factor $\frac{ZN}{p}$ is termed as bearing characteristic number and is a dimensionless number.

The variation of coefficient of friction with the operating values of bearing characteristic number $\frac{ZN}{p}$ is as shown in fig

Sommerfield Number : The Sommerfield number is also a dimensionless number used extensively in the design of bearings.

Mathematically,

$$\text{Sommerfield number} = \frac{ZN}{p} \left(\frac{d}{c} \right)^2$$

For design purpose its value is taken as

$$\frac{ZN}{p} \left(\frac{d}{c} \right)^2 = 14.3 \times 10^5 \text{ to } 1.43 \times 10^9$$

Q. 3. (ii) A stationary journal bearing 47.5 mm in diameter and 38 mm long is bored and lathe turned from a bronze casting. The 1600 rpm journal is 47.3 mm in diameter and is hardened and ground steel. The No. 20 oil has an operating temperature of 77°C. The radial load on the bearing is 51.20 kN. Does the bearing operate under hydrodynamic or marginal lubrication conditions? What is the probable power loss in the bearing due to shear of the oil film?

Aus. Data Given : $W = 51.20 \text{ kN} = 51200 \text{ N}$, $d = 47.5 \text{ mm}$, $l = 38 \text{ mm}$, $N = 1600 \text{ rpm}$,

$t_0 = 77^\circ \text{C}$, No. 20 oil is used.

Let $t_a = 22^\circ \text{C}$, $\frac{l}{d} = \frac{38}{47.5} = 0.8$, $\frac{c}{r} = \frac{0.2}{47.5} = 0.004$

For hardened & ground steel & bronze casting bearing material the value of bearing modulus $\frac{ZN}{p}$ or 5.803.

\therefore The operating bearing number must be

$$3 \times C = 3 \left(\frac{ZN}{p} \right) \\ = 3 \times 5.803 = 17.409$$

(i) $p = \frac{W}{l + d} = \frac{51200}{47.5 \times 38} = 28.36 \text{ N/mm}^2$

(ii) For oil 20 and at 77°C the kinematic viscosity is 35.

Centistoke

$\therefore Z = Z_K \times \rho_{77^\circ \text{C}}$

$$\rho_{77^\circ \text{C}} = \rho_{15} - 0.00063(t - 15) \\ = 0.925 - 0.00063 \times (77 - 15) = 0.88$$

$$Z = 35 \times 10^{-3} \times 0.88$$

$$Z = 31 \text{ centipoise} = 0.031 \text{Ns/m}^2$$

$$\frac{ZN}{p} = \frac{31 \times 10^{-3} \times 1600}{28.36} = 1.748$$

Since the calculated value of $\frac{ZN}{p}$ is less than the selected value of $\frac{ZN}{p}$ so the bearing will not operate under hydrodynamic conditions.

(iii) The coefficient of friction, μ

$$= 0.195 \times 10^6 \left(\frac{ZN}{p} \right) \times \left(\frac{s}{c} \right) \times 10^{-10} + (\Delta f)$$

$$= 0.195 \times 10^6 \times \left(\frac{1.748}{60} \right) \times \frac{1}{0.004} \times 10^{-10} + 0.002$$

$$= 0.000142 + 0.002$$

$$\mu = 0.00214$$

Heat generated

$$H_g = \mu Wv$$

$$= 0.00214 \times 51200 \times \frac{\pi \times 47.5 \times 1600}{60,000}$$

$$= 436 \text{ watt}$$

Heat dissipated

$$= H_d = c'' A = c'' 20 dl$$

$$c'' = K_2 (t_b - t_a) = 4.5 \times 10^{-3}$$

$$= 4.5 \times 10^{-3} \times 47.5 \times 38$$

$$= 8.122 \text{ watt}$$

Since $H_g > H_d$ artificial cooling is required.

Q. 4. Select a single row deep groove ball bearing with the operating cycle listed below, which will have a life of 15000 hrs,

Fraction of Cycle	Type of load	Radial (N)	Thrust (N)	Speed (rpm)	Service factor
1/10	Heavy shocks	2000	1200	400	3.0
1/10	Light shocks	1500	1000	500	1.5
1/5	Moderate shocks	1000	1500	600	2.0
3/5	No shock	1200	2000	800	1.0

Assume radial and axis load factors to be 1.0 and 1.5 respectively and inner race rotates.

Ans. Data given : $L_h = 15000$ hrs

Fraction of cycle	Type of load	Radial N	Thrust N	Speed rpm	Service factor
1/10	Heavy shocks	2000	1200	400	3.0
1/10	Light shocks	1500	1000	500	1.5
1/5	moderate shocks	1000	1500	600	2.0
3/5	No shocks	1200	2000	600	1.0

Radial load factor 1

Axial load factor 1.5

Mean Radial load F_{mr} calculation.

$$F_1 = 2000 \times 3 = 6000, F_2 = 1500 \times 1.5 = 2250,$$

$$F_3 = 1000 \times 2 = 2000, F_4 = 1200 \times 1 = 1200$$

$$N_1 = \frac{1}{10} \times 15000 \times 400 \times 60 = 36 \text{ mor}$$

$$N_2 = \frac{1}{10} \times 15000 \times 500 \times 60 = 45 \text{ mor}$$

$$N_3 = \frac{1}{5} \times 15000 \times 600 \times 60 = 108 \text{ mor}$$

$$N_4 = \frac{3}{5} \times 15000 \times 800 \times 60 = 432 \text{ mor}$$

$$F_{mr} = \left(\frac{F_1^3 N_1 + F_2^3 N_2 + F_3^3 N_3 + F_4^3 N_4}{N} \right)^{1/3}$$

Where $N = N_1 + N_2 + N_3 + N_4$

$$= 36 + 45 + 108 + 432 = 621 \text{ mor}$$

$$F_{mr} = \left(\frac{6000^3 \times 36 + 2250^3 \times 45 + 2000^3 \times 108 + 1200^3 \times 432}{621} \right)^{1/3}$$

$$= \left(\frac{9.899 \times 10^{12}}{621} \right)^{1/3}$$

$$= 2516.716 \text{ N}$$

Mean axial load is

$$F_1 = 1200 \times 3 = 3600$$

$$F_3 = 1500 \times 2 = 3000$$

$$F_2 = 1000 \times 1.5 = 1500$$

$$F_4 = 2000 \times 1 = 2000$$

$$F_{ma} = \left(\frac{3600^3 \times 36 + 1500^3 \times 45 + 3000^3 \times 108 + 2000^3 \times 432}{621} \right)^{1/3}$$

$$= \left(\frac{8.20349 \times 10^{12}}{621} \right)^{1/3}$$

$$= 2364 \text{ N}$$

The equivalent load

$$P = XVF_x + YF_a \quad \{X = 1, Y = 1.5 \text{ Given}\}$$

$$P = 1 \times 1 \times 2516.716 + 1.5 \times 2364 \quad V = 1 \text{ for inner ring Rotates.}$$

$$= 6062.7 \text{ N}$$

$$L_n = \left(\frac{C}{P} \right)^3 \quad \therefore L_n = 621$$

$$C = L_n^{1/3} \times P = 621^{1/3} \times 6062.7$$

$$= 51724.5$$

From the ball bearing table select 55BC03 or 6311. Bearing no. whose C & C₀ are 53940, 41190

$$\frac{F_a}{C_0} = \frac{2364}{41190} = 0.057 = \text{corresponding } l = 0.26$$

$$\frac{F_a}{VF_r} = \frac{2364}{1 \times 2516} = 0.94 \quad \therefore \frac{F_a}{VF_r} > 1$$

Value of X = 0.56

$$Y = 1.71$$

$$P = XVF_r + YF_a$$

$$= 0.56 \times 1 \times 2516 + 1.71 \times 2364$$

$$= 5451 \text{ N}$$

$$L_n = \left(\frac{C}{P} \right)^3 = \left(\frac{53940}{5451} \right)^3$$

$$= (9.895)^3 = 968.9 \text{ mor}$$

$$L_n = 969 \text{ mm} > 621 \text{ mm} \text{ so life is more than the designed life.}$$

So, we can select 55BC03 or 6311 bearing.

Q. 5. Design a close coiled helical compression spring for a service load ranging from 2250N to 2750N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 Mpa and modulus of rigidity $G = 84 \text{ kN/mm}^2$. Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

Ans. (i) $W_1 = 2250 \text{ N}$, $W_2 = 2750 \text{ N}$, $\delta = 6 \text{ mm}$,

$$C = \frac{D}{d} = 5, \tau = 420 \text{ N/mm}^2,$$

$$G = 84 \text{ kN/mm}^2 = 84 \times 10^3 \text{ N/mm}^2$$

Mean diameter of the spring coil = D

Wire diameter of the spring = d

Twisting moment $T = W_2 \times \frac{D}{2} = 2750 \times \frac{5 \times d}{2} = 6875d$

From the stress equation,

$$T = \frac{\pi}{16} \tau d^3 = 6875d$$

$$\frac{\pi}{16} \times 420 \times d^3 = 6875d$$

$$d^2 = \frac{6875 \times 16}{420 \times \pi} \Rightarrow d = 9.13 \text{ mm}$$

For the standard wire dia. from table take $d = 9.49 \text{ mm}$.

$\therefore D = s \times d = s \times 9.49 = 47.45 \text{ mm}$ **Ans.**

$\therefore D_o = D + d = 47.45 + 9.49 = 56.94 \text{ mm}$ **Ans.**

$D_i = D - d = 47.45 - 9.49 = 37.96 \text{ mm}$ **Ans.**

(ii) Number of turns of the spring coil let = i

Since $\delta = 6 \text{ mm}$ for load variation from 2250 to 2750 N

or for $W = 500 \text{ N}$ it is 6mm.

$$\delta = \frac{8WC^3i}{Gd}$$

$$6 = \frac{8 \times 500 \times 5^3 \times i}{84 \times 10^3 \times 9.49} = 0.63i$$

$$i = \frac{C}{0.63} = 9.5 \text{ say } 10 \text{ turns} \quad \text{Ans.}$$

Again for squared and ground ends, the total number of turns,

$$i = 10 + 2 = 12 \quad \text{Ans.}$$

(iii) Free length of spring,

$$L_F = id + \delta_{\max} + 0.15\delta_{\max}$$

Since the compression for 500N is 6mm. \therefore Mean comp., for the mean load 2750 N.

$$\delta_{\max} = \frac{6}{300} \times 2750 = 33\text{mm}$$

$$\begin{aligned} L_F &= 12 \times 9.49 + 33 + 0.15 \times 33 \\ &= 151.83 \text{ say } 152 \text{ mm.} \end{aligned}$$

$$\text{(iv) Pitch of coil} = \frac{\text{free length}}{i-1} = \frac{152}{12-1} = 13.73 \text{ mm Ans.}$$

The spring is shown in fig below.

Q. 6. A semi-elliptical laminated vehicle a spring to carry a load of 6000N is to consist of seven leaves 65 mm wide, two of the leaves extending the full length of the spring. The spring is to be 1.1m in length and attached to the axle by two U-bolts 80mm apart. The bolts hold the central portion of the spring so rigidly that they may be considered equivalent to a bang having a width equal to the distance between the bolts. Assume a design stress for spring material as 350 MPa. Determine : Thickness of leaves : Deflection of spring; Diameter of eye; length of leaves; Radius to which leaves should be initially bent. Sketch the semi elliptical leaf-spring arrangement.

Ans. Data given,

$$2W = 6000\text{N}$$

or

$$W = 3000\text{N}, n = 7, b = 65 \text{ mm}$$

$$n_F = 2, 2L_1 = 1.1 \text{ m or } 1100\text{mm,}$$

$$l = 80\text{mm}, \sigma = 350 \text{ N/mm}^2$$

(i) Thickness of leaves say t ,

$$\text{Effective length} \quad 2L = 2L_1 - l = 1100 - 80 = 1020\text{mm}$$

$$L = \frac{1020}{2} = 510 \text{ mm}$$

$$\text{Number of ground alloy leafs } n_G = n_F = 7 - 2 = 5$$

Assuming that the leaves are not initially stressed, the max. stress σ_F

$$350 = \frac{18 \times WL}{bt^2(2n_G + 3n_F)}$$

$$350 = \frac{18 \times 3000 \times 510}{65 \times t^2(2 \times 5 + 3 \times 2)}$$

$$t^2 = \frac{2628}{250}$$

or

$$t = 8.666 \text{ mm say } 9 \text{ mm}$$

(ii) Deflection of spring,

$$\delta = \frac{12WL^3}{Ebt^3(2n_G + 3n_F)}$$

$$= \frac{12 \times 3000 \times (510)^3}{210 \times 10^3 \times 65 \times 9^3 (2 \times 5 + 3 \times 2)} = 30 \text{ mm}$$

$$\delta = 30 \text{ mm}$$

Ans.

(iii) **Diameter of Eye :** The inner diameter of eye is obtained by considering the pin in the eye in bearing, because the inner diameter of the eye is equal to the diameter of the pin.

Let,

d = Inner dia of the eye

l_1 = Length of the pin which is equal to the width of the eye or leaf = 65 mm

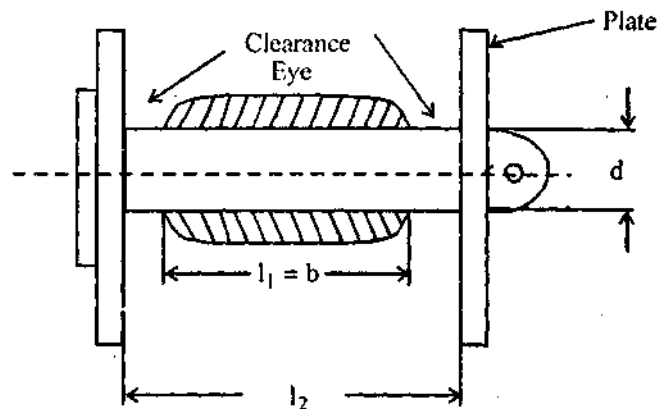
P_b = Bearing pressure on the pin which may be taken as 8 N/mm^2 .

$$W = d \times l_1 \times P_b = \text{Load on the pin}$$

$$3000 = d \times 65 \times 8 = 520d$$

$$d = \frac{3000}{520} = 5.76 \text{ say } 6 \text{ mm} \quad \text{Ans.}$$

Let us consider the bending of the pin.



$$l_2 = l_1 + 2 \times 2 = 65 + 4 = 69 \text{ mm}$$

Max. bending moment on the pin

$$M = \frac{Wl_2}{4} = \frac{3000 \times 69}{4}$$

$$M = 51750 \text{ N-mm}$$

$$Z = \frac{\pi}{32} \times d^3 = 0.0982 d^3$$

$$\sigma_b = \frac{M}{Z} = 80 = \frac{51750}{0.0982 d^3}$$

$$d^3 = \frac{527000}{80} = 6587.32 \approx 18.74 \text{ mm}$$

$$d = \text{say } 2 \text{ mm} \quad \text{Ans.}$$

Taking $\sigma_b = 80 \text{ N/mm}^2$ for pin material.

(iv) Length of leaves

We know that ineffective length of the spring = $l = 80 \text{ mm}$

$$\therefore \text{Length of the smallest leaf} = \frac{\text{Effective length}}{x - 1} + \text{Ineffective length}$$

$$= \frac{1020}{7 - 1} + 80 = 250 \text{ mm} \quad \text{Ans.}$$

$$\text{Length of second leaf} = \frac{1020}{7 - 1} \times 2 + 80 = 420 \text{ mm} \quad \text{Ans.}$$

$$\text{Length of third leaf} = \frac{1020}{7 - 1} \times 3 + 80 = 590 \text{ mm} \quad \text{Ans.}$$

$$\text{Length of fourth leaf} = \frac{1020}{7 - 1} \times 4 + 80 = 760 \text{ mm} \quad \text{Ans.}$$

$$\text{Length of fifth leaf} = \frac{1020}{7 - 1} \times 5 + 80 = 930 \text{ mm} \quad \text{Ans.}$$

$$\text{Length of sixth leaf} = \frac{1020}{7 - 1} \times 6 + 80 = 1100 \text{ mm} \quad \text{Ans.}$$

The sixth & seventh leaves are full length leaves and the seventh leaf will act as a master leaf.

\therefore Length of the master leaf

$$= 2L_1 + \pi(d + t) \times 2$$

$$= 1100 + \pi(20 + 9) \times 2$$

$$= 1282.2 \text{ mm} \quad \text{Ans.}$$

(v) Radius to which the leaves should be initially bent.

Let R = Radius to which the leaves should be initially bent

y = Camber of the spring.

We know that,

$$y(2R - y) = (L_1)^2$$

$$\therefore 30(2R - 30) = (550)^2$$

$$\text{Or } R = \frac{(550)^2 / 30 + 30}{2} = 5056.65 \text{ mm} \quad \text{Ans.}$$

$$\therefore y = \delta \text{ which is camber of the spring.}$$

Q. 7. A gear drive is required to transmit a maximum power of 22.5 kW. The velocity ratio is 1 : 2 and rpm of the pinion is 200. The approximate center distance between the shafts may be taken as 600mm. The teeth has 20° stub involute profiles. The static stress for the gear material (cast iron) may be taken as 60 MPa and face width as 10 times the module. Find the module, face width and number of teeth on each gear. Check the design for dynamic factor in the Buckingham equation may be taken as 80 and the material combination factor for the wear as 1.4.

Ans. Data given :

$$P_i = 22.5 \text{ kW}$$

$$\sigma_g = \sigma_p = 60 \text{ N/mm}^2$$

$$G = 1 : 2$$

$$b = 10m$$

$$N_p = 200 \text{ rpm}$$

$$\text{dynamic factor} = 80$$

$$a = 600 \text{ mm}$$

$$\text{wear factor} = 1.4$$

$$\phi = 20^\circ \text{ stub involute}$$

As both the gear and pinion is of same material so the pinion is a weaker one.

$$\text{Angular velocity ratio} = \frac{N_p}{N_g} = \frac{2}{1} = \frac{d_g}{d_p} \cdot \frac{Z_p}{Z_g}$$

$$a = \frac{d_p + d_g}{2} = d_p + d_g = 600 \times 2 = 1200 \text{ mm}$$

$$d_g = 2\phi_p \text{ from these two relations}$$

$$d_p = 400$$

$$d_g = 800$$

Since the design is based on pinion.

$$V_p = \frac{\pi d_p N_p}{60,000} = \frac{\pi \times 400 \times 200}{60,000} = 1.333 \text{ m/sec.}$$

Tangential load on the tooth to be transmit

$$F_t = \frac{1000 P \times C_s}{v} = \frac{1000 \times 22.5 \times 1}{1.333} = 16879.2 \text{ N}$$

Assuming service factor $C_s = 1$

$$C_v = \frac{4.5}{4.5 + 1.333} = 0.77 \text{ velocity factor.}$$

Assuming $y = 0.12$ of 20° stub tooth from table C_y lies between (0.999 and 0.17)

From the beam strength equation,

$$F_t = \sigma_d C_v b \pi y m$$

$$16879.2 = 60 \times 0.77 \times 10 \times m \times \pi \times 0.12 \times m$$

$$16879.2 = 174.169 m^2$$

$$96.912 = m^2$$

$$m = 9.844 \text{ mm} \quad \text{choose } m = 10 \text{ mm as standard}$$

$$Z_p = \frac{400}{m} = \frac{400}{10} = 40$$

$$Z_g = \frac{800}{m} = \frac{800}{10} = 80$$

And corresponding centre distance

$$a = \frac{m Z_p + m Z_g}{2}$$

$$a = \frac{10 \times 40 + 10 \times 80}{2} = 600 \text{ mm as desired.}$$

$$b = 10m = 10 \times 10 = 100 \text{ mm}$$

\therefore Dimensions of gear pair

$$m = 10, Z_p = 40, Z_g = 80, b = 100 \text{ mm}$$

$$d_p = 400, d_g = 800, a = 600$$

$$h_a = 1m = 10, h_f = 1.25m = 12.5 \text{ mm}$$

Check for design

The Buckingham's equation for the dynamic load in tangential direction is given by

$$F_d = \frac{21v(cb + F_t)}{21v + \sqrt{cb + F_t}}$$

$$= \frac{21 \times 1.33(80 \times 100 + 16879)}{21 \times 1.33 + \sqrt{80 \times 100 + 16879}}$$

$$= \frac{694870.47}{185.66}$$

$$= 3742.688 \text{ N}$$

$$F_{\text{eff}} = \frac{K_a K_m F_t}{C_v} = \frac{1 \times 1 \times 16879}{0.77} = 21920.779 \text{ N}$$

Since $F_d < F_{\text{eff}}$ so gear is safe against bending

Wear load $F_w = d_p b Q k$

$$Q = \frac{2Z_g}{Z_g + Z_p} = \frac{2 \times 80}{80 + 40} = 1.33$$

$$F_w = 400 \times 100 \times 1.33 \times 1.4$$

$$= 74666.66 \text{ N}$$

Since the wear load $F_w > F_d$ so design is safe against wear load.

Q. 8. Design 20° involute worm and gear to transmit 10kW with worm rotating at 1400 rpm and to obtain a speed reduction of 12 : 1. The distance between the shafts is 225mm.

Ans. Data given,

$$P_i = 10 \text{ kW} = 10,000 \text{ W}, \phi_n = 20^\circ \text{ involute}$$

$$n_w = 1400 \text{ rpm} \quad G = 12.1$$

$$a = 225 \text{ mm}$$

Assumption for Material Selection :

(i) For Worm Gear : Phosphor bronze with $\sigma_{ag} = 240 \text{ N/mm}^2$

For Worm : Case hardened and ground alloy steel,

(ii) Assume $K_a = 1.5$ $b = 0.73 d_w$

$$K_v = \frac{6}{6 + V_g}$$

(iii) For case hardened ground steel worm and phosphor bronze worm gears $\mu = 0.3$.

(iv) For $G = 12 (7 < G < 15)$ $Z_w = 3$ (selected)

Number of teeth on worm gear and module.

$$G = \frac{Z_g}{Z_w}$$

$$12 = \frac{Z_g}{3}$$

$$Z_g = 38$$

Now,

$$a = \frac{d_g + d_w}{2} = \frac{mz_g + m_q}{2}$$

$$225 = \frac{m(z_g + q)}{2}$$

$$450 = m(36 + q)$$

$$m = \frac{450}{(36 + q)}$$

The value of module 'm' and diametral quotient q are selected as follows :

q	6	7	8	9	10	11	12	13
m	10.71	10.48	10.22	10	9.78	9.57	9.37	9.16
Remarks	Not feasible	Not feasible	Not feasible	OK	Not feasible	Not feasible	Not feasible	Not feasible

$$q = 9$$

$$m = 10$$

(i) Lead angle

$$\tan \lambda = \frac{Z_w}{q} = \frac{3}{9}$$

$$\lambda = \tan^{-1} \left(\frac{Z_w}{q} \right) = \tan^{-1} \left[\frac{3}{9} \right] = 18.43^\circ$$

(ii) For $\lambda = 18.43^\circ$, hardened steel worm and phosphor bronze worm gear

$$K = 0.687$$

(iii) Dimension of worm and worm gear

$$m = 10$$

$$Z_w = 3$$

$$Z_g = 36$$

$$d_w = m + q = 10 + 9 = 90 \text{ mm}$$

$$d_g = mz_g = 10 \times 36 = 360 \text{ mm}$$

$$P_a = \pi m = \pi \times 10 = 31.47 \text{ mm}$$

$$L = P_a \times Z_w = 31.47 \times 3 = 94.24 \text{ mm}$$

$$\lambda = 18.43^\circ$$

$$b = 0.73 d_w = 0.73 \times 90 = 65.7 \text{ mm}$$

$$L_w = \pi m \left[4.5 + \frac{Z_g}{50} \right] \text{ for triple \& quadruple threads}$$

$$= \pi \times 10 \left[4.5 + \frac{36}{50} \right] = 164 \text{ mm}$$

$$h_a = 1m = 10 \text{ mm}$$

$$h_f = 1.2m = 12 \text{ mm}$$

$$a = \frac{d_w + d_g}{2} = \frac{90 + 360}{2} = 225 \text{ mm}$$

Hence, the designation of worm gear pair is 3/36/9/10.

(iv) Beam strength of worm tooth.

$$\sigma_{\text{disp}} = \frac{\sigma_{\text{ug}}}{3} = \frac{240}{3} = 80 \text{ N/mm}^2$$

For 20° full depth involute,

$$Y = 0.484 - \frac{2.87}{Z_g} = 0.484 - \frac{2.87}{36} = 0.563$$

$$F_b = \sigma_{\text{dg}} \times b \times m \times Y \cos \lambda$$

$$= 80 \times 65.7 \times 10 \times 0.563 \times \cos 18.43$$

$$F_b = 28,073.5 \text{ N}$$

(v) Wear strength of worm gear tooth

$$F_w = d_g b k$$

$$= 360 \times 65.7 \times 0.687$$

$$= 16,248.9 \text{ N}$$

As $F_b < F_w$ worm gear is weaker in bending, and hence it should be checked for safety against bending failure.

(vi) Effective load on worm gear tooth

$$\phi_v = \tan^{-1} \{ \mu_v \} = \tan^{-1} \left\{ \frac{\mu}{\cos \phi_n} \right\} = \tan^{-1} \left[\frac{0.03}{\cos 20} \right] = 1.8285^\circ$$

$$\eta = \frac{\tan \lambda}{\tan(\phi_v + \lambda)} = \frac{\tan 18.43}{\tan[1.8285 + 18.43]} = \frac{0.333}{0.369} = 90.24\%$$

The output power is given by,

$$P_o = P_i \times \eta = 10,000 \times 0.9 = 9000 \text{ W}$$

$$n_g = \frac{n_w}{G} = \frac{1400}{12} = 116.66 \text{ rpm}$$

$$V_g = \frac{\pi d_g n_g}{60 + 100} = \frac{\pi \times 360 \times 116.66}{60 \times 1000} = 2.2 \text{ m/sec.}$$

$$(F_g)_t = \frac{P_o}{V_g} = \frac{9000}{2.2} = 4092.5 \text{ N}$$

$$K_v = \frac{6}{6 + V_g} = \frac{6}{6 + 2.2} = 0.73$$

$$F_{eff} = \frac{K_a (F_g)_t}{K_v} = \frac{1.5 \times 4092.5}{0.73} = 8409.24 \text{ N}$$

(vii) Available FOS against bending failure,

$$N_{fb} = \frac{F_b}{F_{eff}} = \frac{28073}{8409} = 3.3$$

(viii) Available FOS against pitting failure,

$$N_{fw} = \frac{F_w}{F_{eff}} = \frac{16248}{8409} = 1.93$$

Hence, worm gear reduces is safe against pitting failure. **Ans.**