

8. (a) Define an algebraic structure with the help of examples. Also, give the necessary conditions for an algebraic structure to be a group, semi-group. 8
- (b) State and prove Lagrange's theorem. 7
9. (a) Define monoid and give examples. Prove that identify elements is unique in monoid. 7
- (b) Define the following by giving *two* examples in each case : 8
- (i) Group
- (ii) Cyclic Group
- (iii) Normal Sub-group

Roll No. ....

3026

B. Tech. 3rd Semester (Civil Engg.)  
Examination – December, 2022

MATHEMATICS - III

Paper : BSC-MATH-205-G

Time : Three Hours ]

[ Maximum Marks : 75

*Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.*

Note : Attempt *five* questions in all, selecting *one* question from each Unit. Question No. 1 is *compulsory*. All questions carry equal marks.

1. (a) How are the partial differential equations classified ? Give an example for each type.
- (b) Write Regula Falsi method to solve non-linear equations.
- (c) State Newton's backward interpolation formula.
- (d) Find the Laplace transform of  $e^t \sin 4t \cos 2t$ .
- (e) Solve :  $z = px + qy + pq$
- (f) Define Permutation. 2.5 × 6 = 15

UNIT - I

2. (a) Solve the following differential equation : 7.5

$$(mz - ny)p + (nx - lz)q = ly - mx$$

- (b) Solve by Charpit's method :  $z^2 = pqxy$  7.5

3. (a) A string is stretched and fastened to two points  $x = 0$  and  $x = l$ . Motion is started by displacing the string in the form  $y = a \sin\left(\frac{\pi x}{l}\right)$  from which it is released at time  $t = 0$ . Show that the displacement is given by

$$y(x, t) = a \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi ct}{l}\right) \quad 7.5$$

- (b) Using method of separation of variables, solve  $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ , where  $u(x, 0) = 6e^{-3x}$ . 7.5

UNIT - II

4. (a) Find a real root of the equation  $x^3 - x - 9 = 0$  by bisection method current to four decimal places. 7.5

- (b) Find the cubic polynomial which takes the following values :

$x$	0	1	2	3
$f(x)$	1	2	1	10

Hence or otherwise evaluate  $f(4)$ . 7.5

5. (a) Given the values :

$x$	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

Evaluate  $\log_{10} 301$  by using Newton's divided difference formula 7.5

- (b) Evaluate :  $\int_0^1 \frac{4x}{1+x^2}$  using :

- (i) Trapezoidal rule taking  $h = \frac{1}{4}$

- (ii) Simpson's rule taking  $h = \frac{1}{6}$  7.5

UNIT - III

6. (a) Using Convolution theorem, find inverse Laplace transformation of  $\frac{s}{(s^2 + a^2)^2}$ . 7.5

- (b) Solve  $\frac{d^2x}{dt^2} + 9x = \cos 2t$ ,  $x(0) = -1$ ,  $x'(0) = 2$ , using Laplace transform. 7.5

7. (a) (i) Evaluate :  $\int_0^\infty \left( \frac{e^{-t} - e^{-3t}}{t} \right) dt$  4

- (ii) Find Laplace Inverse of  $\tan^{-1} \left( \frac{2}{s^2} \right)$  4

- (b) Find Laplace inverse transform of  $\log \frac{s(s+1)}{(s^2+4)}$ . 7