

Roll No. ....

**3057**

**B. Tech. 3rd Semester (ME)  
Examination – December, 2022**

**MATHEMATICS - III (PDE, Probability & Statistics)**

**Paper : BSC-ME-203-G**

**Time : Three Hours ]**

**[ Maximum Marks : 75**

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*Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.*

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**Note :** Attempt *five* questions in all, selecting *one* question from each Unit. Question No. 1 is *compulsory*. All questions carry equal marks.

1. (a) Define Partial Differential Equation with example. 2.5
- (b) Explain Homogeneous Partial Differential Equation of Higher Order with example. 2.5
- (c) Define skewness and write the formula of calculating skewness. 2.5
- (d) What do you mean by Sampling ? Explain its purpose and significance. 2.5

- (e) Define Binomial Distribution and mention its assumptions. 2.5
- (f) A random variable 'X' has the following probability density function. Calculate the value of k : 2.5

|      |   |    |    |    |    |     |     |
|------|---|----|----|----|----|-----|-----|
| X    | 0 | 1  | 2  | 3  | 4  | 5   | 6   |
| p(X) | k | 3k | 5k | 7k | 9k | 11k | 13k |

### UNIT - I

2. (a) Solve linear partial differential equation : 7.5

$$(mz - ny)p + (nx - lz)q = ly - mx$$

- (b) Solve Homogeneous partial differential equation : 7.5

$$(D^2 + 2DD' + D'^2)z = e^{2x+3y}$$

3. (a) Solve Non-homogeneous partial differential equation : 7.5

$$(D^2 + DD' + D' - 1)z = \sin(x + 2y)$$

- (b) Obtain D'Alembert's solution of wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \left( \frac{\partial^2 u}{\partial t^2} \right) \text{ given that initial deflection}$$

$$u(x, 0) = f(x) \text{ and initial velocity } \left( \frac{\partial u}{\partial t} \right)_{t=0} = g(x). \quad 7.5$$

### UNIT - II

4. (a) Solve the following equation using method of separation of variables  $4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$  given that

$$u(0, y) = 3e^{-y} - e^{-5y}. \quad 7.5$$

(b) Solve one dimensional Heat Equation  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  with

boundary conditions  $u(x, 0) = 3 \sin n\pi x$ ;  $u(0, t) = 0$   
and  $u(1, t) = 0$ . 7.5

5. Write down the Laplace's equation in plane in two dimensional polar co-ordinates  $(r, \theta)$  and find its solution using method of separation of variables. 15

### UNIT - III

6. (a) If four coins are tossed together. Find the expectation of number of Heads. 7.5

(b) State and prove Chebyshev's inequality. 7.5

7. (a) (i) Define continuous random variable and state its properties. 2.5

(ii) A function is defined as follows :

$$\begin{aligned} f(x) &= 0 & x < 2 \\ &= \frac{1}{18}(2x+3) & 2 \leq x \leq 4 \\ &= 0 & x > 4 \end{aligned}$$

Show that it is a density function. 5

(b) In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total of their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random and is found to be defective. What is the probability that it was manufactured by machine B? 7.5

## UNIT - IV

8. (a) Students of a class were given mechanical aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What percent of students scored :

(i) More than 60 marks,

(ii) Less than 56 marks ? 7.5

(b) Ten participants in a contest are ranked by two judges as follows :

|     |   |   |   |    |   |   |   |    |   |   |
|-----|---|---|---|----|---|---|---|----|---|---|
| $x$ | 1 | 6 | 5 | 10 | 3 | 2 | 4 | 9  | 7 | 8 |
| $y$ | 6 | 4 | 9 | 8  | 1 | 2 | 3 | 10 | 5 | 7 |

Calculate rank correlation coefficient. 7.5

9. (a) A manufacturer claims that only 4% of his products supplied by him are defective. A random sample of 600 products contained 36 defectives. Test the claim of manufacturer. 7.5

(b) The following table gives the number of accidents that took place in an industry during various days of the week. Apply Chi-square test to verify whether accidents are uniformly distributed over the week or not : 7.5

| Day       | Mon. | Tue. | Wed. | Thu. | Fri. | Sat. |
|-----------|------|------|------|------|------|------|
| Accidents | 14   | 18   | 12   | 11   | 15   | 14   |